Semi-orthogonal linear area magic squares of order 4

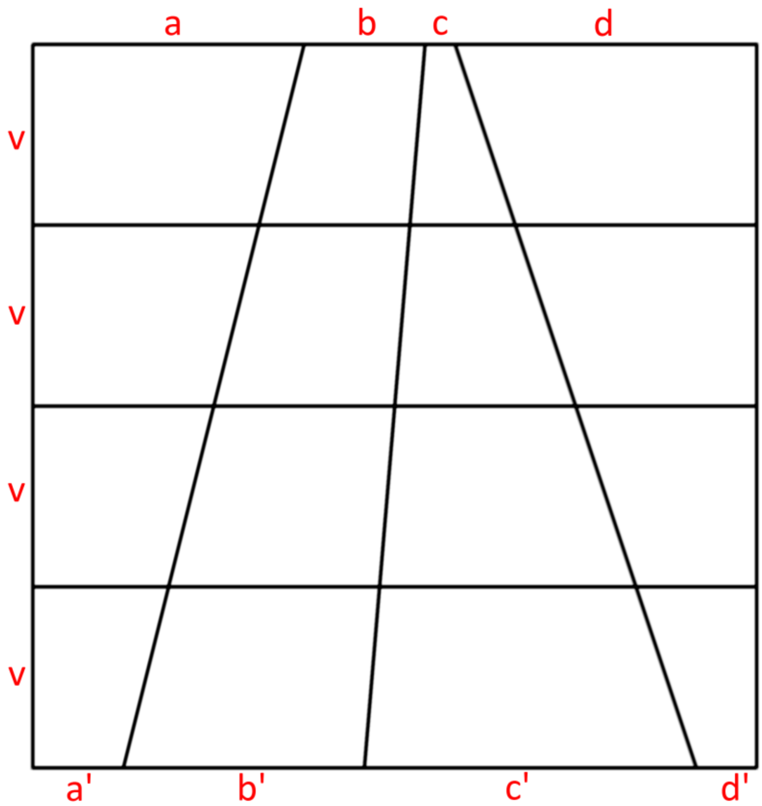
Part 2

How to draw a semi-orthogonal linear area magic square?

Walter Trump, 2017-02-05

10 Which parameters are necessary?

The necessary parameters are the distances of the intersections on the borders. In the graphic the names of these distances are shown in red.



A complete description of an L-AMS of order 4 is given by the following table.

S	a	b	c	d
v	A	B	C	D
v	A_2	B_2	C_2	D_2
v	A_3	B_3	C_3	D_3
v	A_4	B_4	C_4	D_4
	a‘	b‘	c‘	d‘

Magic Sum S = A + B + C + D

11 How can we determine the distances?

We first calculate auxiliary values v_0 , a_0 , b_0 , c_0 and d_0 ,
which generally are greater than the final results.

$v_0 = S / 2$

The other distances can be calculated from the entries of the first two rows.

$a_0 = 2(A + A_2) - v_0$, $b_0 = 2(B + B_2) - v_0$, $c_0 = 2(C + C_2) - v_0$, $d_0 = 2(D + D_2) - v_0$

Then we determine the greatest common divisor of v_0 , a_0 , b_0 , c_0 and d_0 .

If $f = \gcd(v_0 , a_0 , b_0 , c_0 , d_0)$ then ...

$v = v_0 / f$, $a = a_0 / f$, $b = b_0 / f$, $c = c_0 / f$, $d = d_0 / f$.

$a' = 2v - a$ $b' = 2v - b$ $c' = 2v - c$ $d' = 2v - d$

In order to achieve that the square area is equal to 4S, we use a size factor ***e***.

$(4\, e \cdot v)^2 = 4S \;\Rightarrow\; e^2 = S / (4 \cdot v^2) \;\Rightarrow\; e = \frac{1}{2v}\sqrt{S}$

The true value of v is $v_{true} = u \cdot v$.

But in most cases the exact size is not important.

12 Example for the calculation of the distances

Magic sum S = 84 Entries:

18	33	6	27
20	25	16	23
22	17	26	19
24	9	36	15

$v_0 = S / 2 = 84 / 2 = 42$

$a_0 = 2(A + A_2) - v_0 = 2 \cdot (18 + 20) - 42 = 34$

$b_0 = 2(B + B_2) - v_0 = 2 \cdot (33 + 25) - 42 = 74$

$c_0 = 2(C + C_2) - v_0 = 2 \cdot (6 + 16) - 42 = 2$

$d_0 = 2(D + D_2) - v_0 = 2 \cdot (27 + 23) - 42 = 58$

All values are multiples of 2: $f = \mathrm{gcD}(v_0 , a_0 , b_0 , c_0 , d_0) = 2$

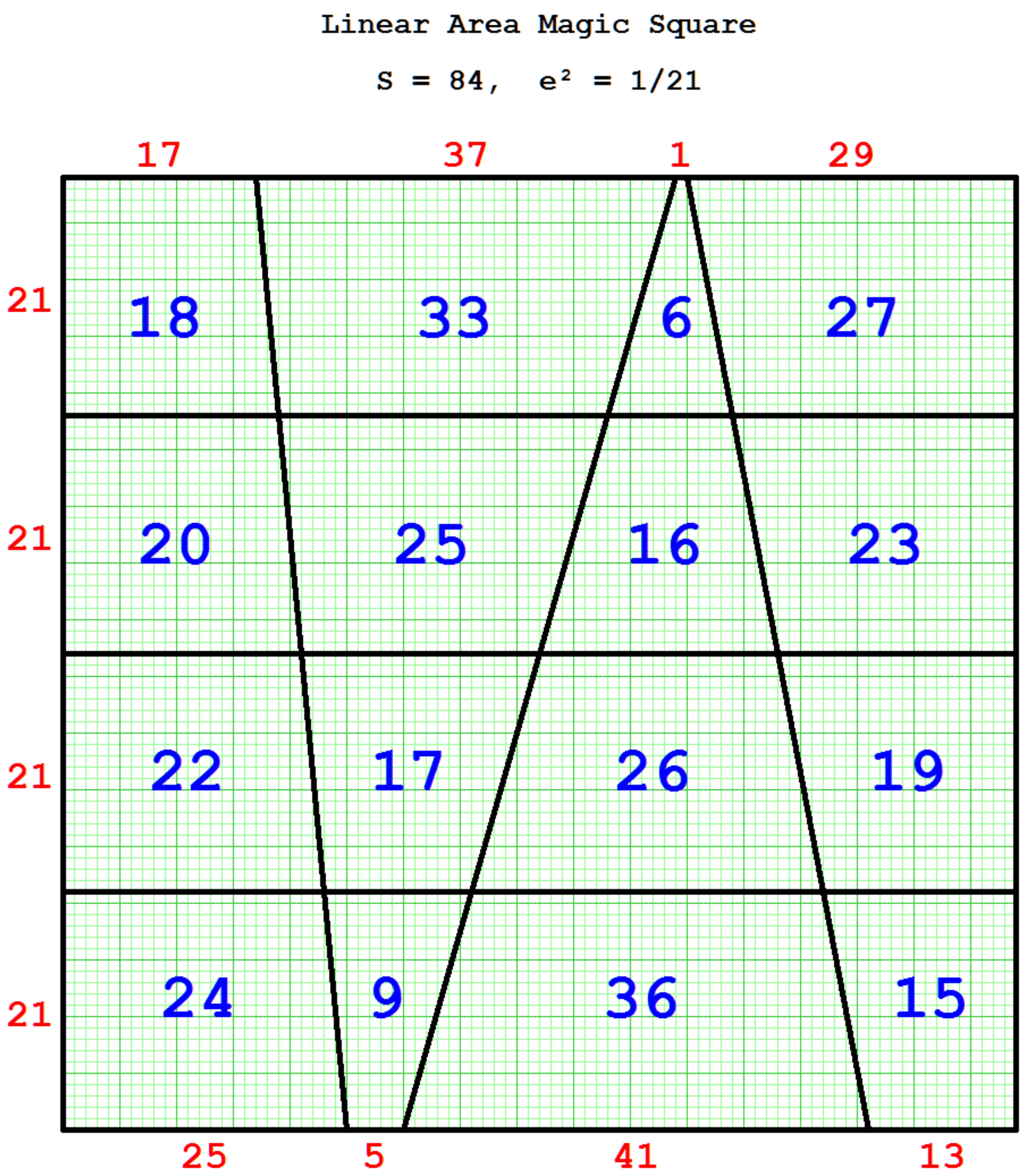
$v = v_0 / f = 42 / 2 = 21$

$a = a_0 / 2 = 17$	$a' = (2v - a) = 42 - 17 = 25$
$b = b_0 / 2 = 37$	$b' = (2v - b) = 42 - 37 = 5$
$c = c_0 / 2 = 1$	$c' = (2v - c) = 42 - 1 = 41$
$d = d_0 / 2 = 29$	$d' = (2v - d) = 42 - 29 = 13$
sum = 4v = 84	sum = 4v = 84

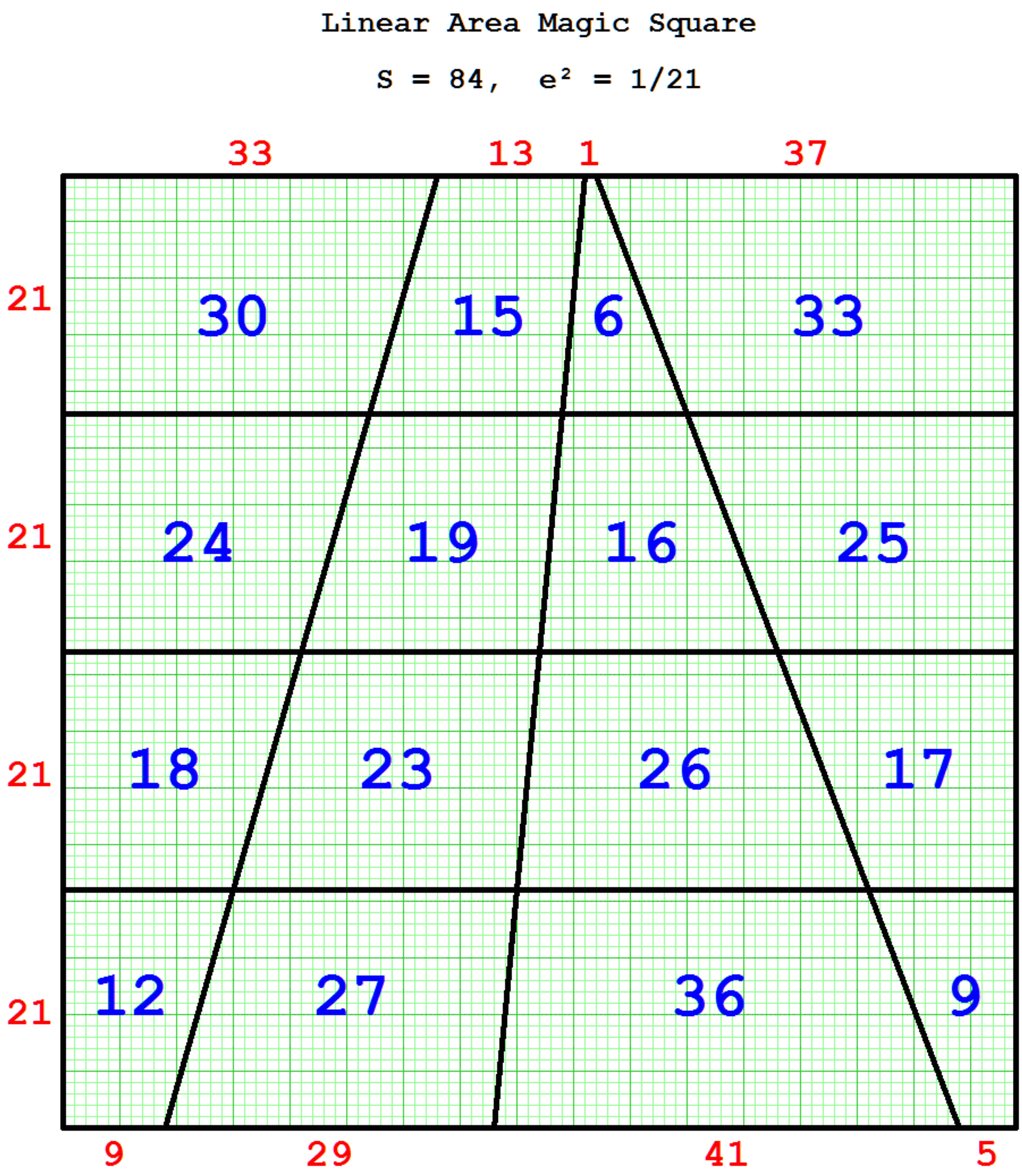
84	17	37	1	29
21	**18**	**33**	**6**	**27**
21	**20**	**25**	**16**	**23**
21	**22**	**17**	**26**	**19**
21	**24**	**9**	**36**	**15**
	25	5	41	13

(Size factor $e^2 = \frac{S}{4v^2} = \frac{84}{4 \cdot 21^2} = \frac{1}{21}$)

13 Drawing of the example with S = 84

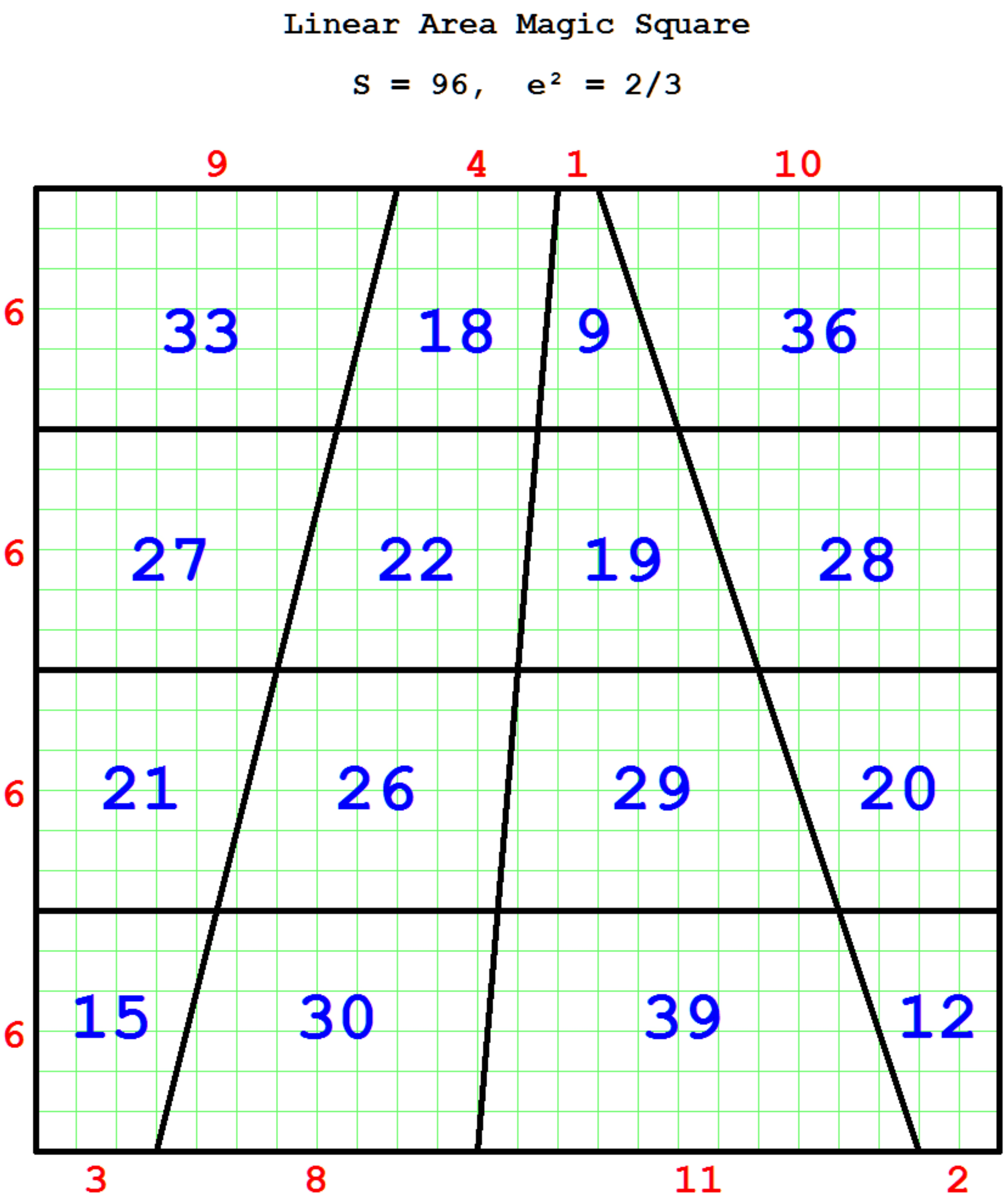


14 Second solution with S = 84

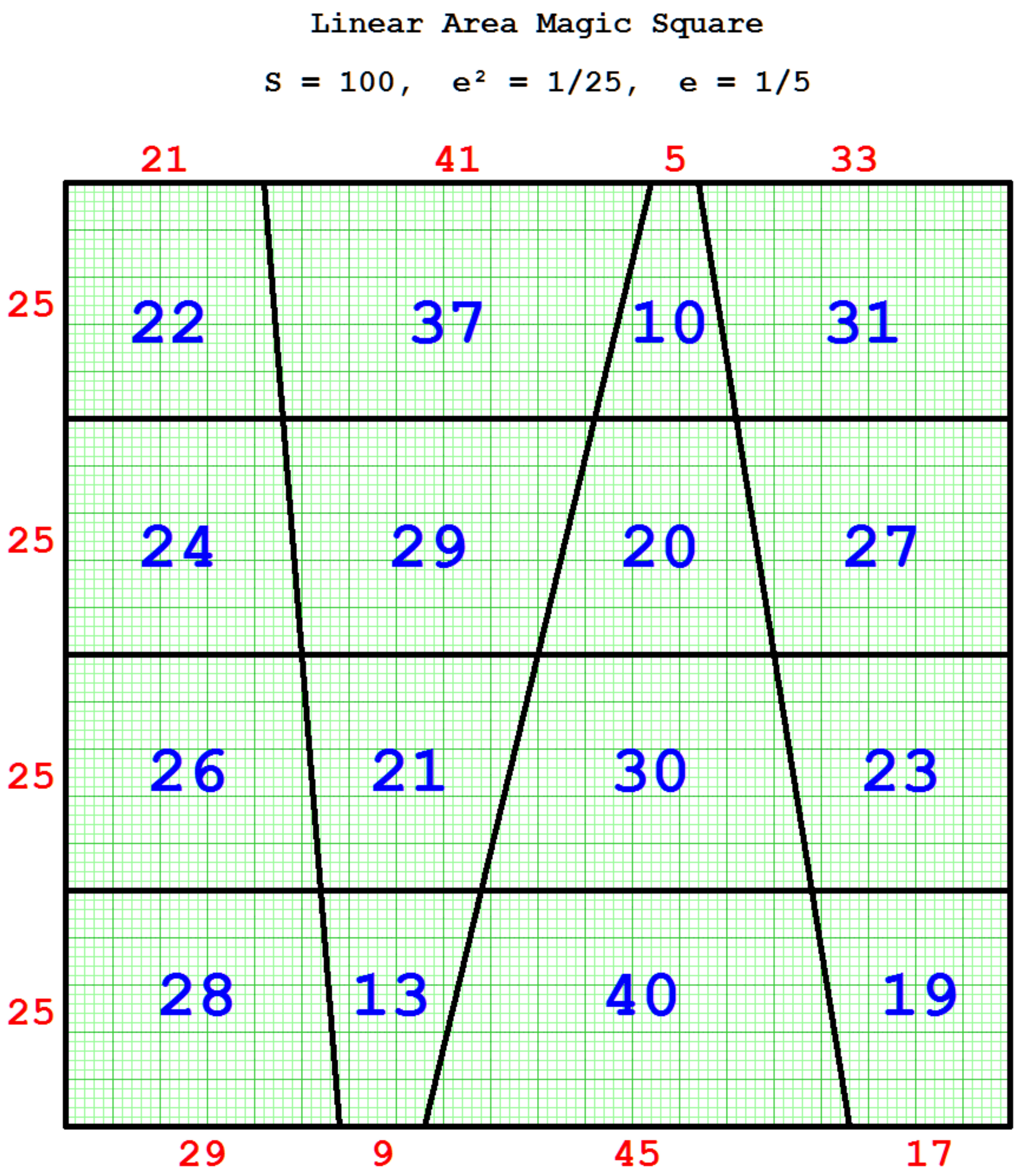


All graphics were produced automatically by a computer program.

15 First found L-AMS – 2017-01-14



16 L-AMS found by Hans-Bernhard Meyer – 2017-01-16



17 L-AMS with magic sum 144